Supplement to "Designing $\overline{X}$ Charts for Known Autocorrelations and Unknown Marginal Distribution"

Huifen Chen and Yuyen Cheng
Department of Industrial and Systems Engineering, Chung-Yuan University
200 Chung-Pei Rd., Chungli, 32023, TAIWAN

March 5, 2008

This supplement shows the standard-error estimates for $\widehat{\text{ARL}}_0$ and $\widehat{\text{ARL}}_\delta$ (the estimates of the in-control and out-of-control average run lengths) in Tables 2, 3, and 6 of the main paper. The standard-error estimates are listed in Tables S1 to S3. Also shown here is the comparison of the R&W, tuned R&W, DFTC, and modified-Method-2 charts for ARTA(1) processes with exponential marginal distribution. Table S4 shows the $\widehat{\text{ARL}}_0$ and $\widehat{\text{ARL}}_\delta$ values for the four charts and Table S5 shows the standard-error estimates for $\widehat{\text{ARL}}_0$ and $\widehat{\text{ARL}}_\delta$ in Table S4. Below is a summary of these five tables:

Table S1: Standard errors of $\widehat{\text{ARL}}_0$ and $\widehat{\text{ARL}}_\delta$ in Table 2 of the main paper for Method 1, Method 2, and the true optimal solution, where the data process is ARTA(1) with t_{10} marginal distribution

Table S2: Table S2: Standard errors of $\widehat{\text{ARL}}_0$ and $\widehat{\text{ARL}}_\delta$ in Table 3 of the main paper for modified Method 1, modified Method 2, and the true optimal solution, where the data process is ARTA(1) with t_{10} marginal distribution

Table S3: Table S3: Standard errors of $\widehat{\text{ARL}}_0$ and $\widehat{\text{ARL}}_\delta$ in Table 6 of the main paper for the R&W, tuned R&W, DFTC, and modified-Method-2 charts, where the data process is ARTA(1) with t_{10} marginal distribution

Table S4: Table S4: Comparisons of the R&W, tuned R&W, DFTC, and modified-Method-2 charts for ARTA(1) processes with exponential marginal distribution, $\rho_1 = 0$, 0.25, 0.5, 0.7, 0.9, and $L = 1000$ (Note: The table format is the same as that of Table 6 in the main paper.)

Table S5: Table S5: Standard errors of $\widehat{\text{ARL}}_0$ and $\widehat{\text{ARL}}_\delta$ in Table S4.

Table S1: Standard errors (se's) of $\widehat{\text{ARL}}_0$ and $\widehat{\text{ARL}}_\delta$ in Table 2 for ARTA(1) processes with t_{10} marginal distribution, $\rho_1 = 0$, 0.25, 0.5, 0.7, 0.9, and $L = 1000$

		Method 1		Method 2		true optimal solution
ρ_1	δ	se of $\widehat{\text{ARL}}_0$	se of $\widehat{\text{ARL}}_\delta$	se of $\widehat{\text{ARL}}_0$	se of $\widehat{\text{ARL}}_\delta$	se of $\widehat{\text{ARL}}_\delta$
0	0.25	3	0.3			0.1
	0.5	3	0.09			0.03
	0.75	3	0.05	same as for		0.03
	1	3	0.03	Method 1		0.02
	1.5	3	0.01			0.01
	2	2	0.01			< 0.01
	4	1	< 0.01			< 0.01
0.25	0.25	3	0.4	3	0.4	0.1
	0.5	3	0.1	3	0.1	0.1
	0.75	3	0.07	3	0.07	0.05
	1	3	0.04	3	0.04	0.03
	1.5	3	0.02	3	0.02	0.02
	2	2	0.01	2	0.01	0.01
	4	1	< 0.01	1	< 0.01	< 0.01
0.5	0.25	3	0.6	3	0.6	0.2
	0.5	3	0.2	3	0.2	0.1
	0.75	3	0.1	3	0.1	0.04
	1	3	0.07	3	0.07	0.03
	1.5	3	0.04	3	0.03	0.02
	2	2	0.03	2	0.02	0.01
	4	1	< 0.01	1	< 0.01	< 0.01
0.7	0.25	3	0.9	3	0.9	0.3
	0.5	3	0.3	3	0.3	0.1
	0.75	3	0.2	3	0.2	0.1
	1	3	0.1	3	0.1	0.09
	1.5	3	0.07	3	0.06	0.04
	2	1	0.07	2	0.04	0.03
	4	1	0.01	1	< 0.01	< 0.01
0.9	0.25	3	2	3	2	0.6
	0.5	3	0.9	3	0.9	0.3
	0.75	3	0.5	3	0.5	0.1
	1	3	0.4	3	0.3	0.1
	1.5	2	0.4	3	0.2	0.1
	2	2	0.2	3	0.1	0.07
	4	2	0.01	1	0.01	0.01

Table S2: Standard errors (se's) of $\widehat{\text{ARL}}_0$ and $\widehat{\text{ARL}}_\delta$ in Table 3 for ARTA(1) processes with t_{10} marginal distribution, $\rho_1 = 0$, 0.25, 0.5, 0.7, 0.9, and $L = 1000$

		modified Method 1		modified Method 2		true optimal solution
ρ_1	δ	se of $\widehat{\text{ARL}}_0$	se of $\widehat{\text{ARL}}_\delta$	se of $\widehat{\text{ARL}}_0$	se of $\widehat{\text{ARL}}_\delta$	se of $\widehat{\text{ARL}}_\delta$
0	0.25	3	0.3			0.1
	0.5	3	0.08			0.03
	0.75	3	0.02	same as for the		0.03
	1	3	< 0.01	modified Method 1		0.02
	1.5	3	< 0.01			0.01
	2	3	< 0.01			< 0.01
	4	3	< 0.01			< 0.01
0.25	0.25	3	0.4	3	0.4	0.1
	0.5	3	0.1	3	0.1	0.1
	0.75	3	0.05	3	0.05	0.05
	1	3	0.01	3	0.01	0.03
	1.5	3	< 0.01	3	< 0.01	0.02
	2	3	< 0.01	3	< 0.01	0.01
	4	3	< 0.01	3	< 0.01	< 0.01
0.5	0.25	3	0.6	3	0.6	0.2
	0.5	3	0.2	3	0.2	0.1
	0.75	3	0.1	3	0.1	0.04
	1	3	0.05	3	0.05	0.03
	1.5	3	0.01	3	0.01	0.02
	2	3	< 0.01	3	< 0.01	0.01
	4	3	< 0.01	3	< 0.01	< 0.01
0.7	0.25	3	0.9	3	0.9	0.3
	0.5	3	0.3	3	0.3	0.1
	0.75	3	0.2	3	0.2	0.1
	1	3	0.1	3	0.1	0.09
	1.5	3	0.03	3	0.03	0.04
	2	3	0.01	3	0.01	0.03
	4	3	< 0.01	3	< 0.01	< 0.01
0.9	0.25	3	2	3	2	0.6
	0.5	3	0.9	3	0.9	0.3
	0.75	3	0.5	3	0.5	0.1
	1	3	0.4	3	0.3	0.1
	1.5	3	0.2	3	0.2	0.1
	2	3	0.08	3	0.08	0.07
	4	3	< 0.01	3	< 0.01	0.01

Table S3: Standard errors (se's) of $\widehat{\text{ARL}}_0$ and $\widehat{\text{ARL}}_\delta$ in Table 6 for ARTA(1) processes with t_{10} marginal distribution, $\rho_1 = 0$, 0.25, 0.5, 0.7, 0.9, and $L = 1000$

					Modified Method 2	
ρ_1	δ	R&W	Tuned R&W	DFTC	se of $\widehat{\text{ARL}}_0$	se of $\widehat{\text{ARL}}_\delta$
		se of $\widehat{\text{ARL}}_0 = 0.8$	se of $\widehat{\text{ARL}}_0 = 3$	se of $\widehat{\text{ARL}}_0 = 3$		
	0.25	—	0.3	0.2	3	0.3
	0.5	—	0.03	0.06	3	0.08
0	0.75	—	< 0.01	0.03	3	0.02
	1	—	< 0.01	0.02	3	< 0.01
	1.5	—	< 0.01	< 0.01	3	< 0.01
	2	—	< 0.01	< 0.01	3	< 0.01
	4	—	< 0.01	< 0.01	3	< 0.01
		se of $\widehat{\text{ARL}}_0 = 2$	se of $\widehat{\text{ARL}}_0 = 3$	se of $\widehat{\text{ARL}}_0 = 4$		
	0.25	—	0.5	0.3	3	0.4
	0.5	—	0.09	0.09	3	0.1
0.25	0.75	—	0.01	0.04	3	0.05
	1	—	< 0.01	0.03	3	0.01
	1.5	—	< 0.01	0.01	3	< 0.01
	2	—	< 0.01	< 0.01	3	< 0.01
	4	—	< 0.01	< 0.01	3	< 0.01
		se of $\widehat{\text{ARL}}_0 = 3$	se of $\widehat{\text{ARL}}_0 = 3$	se of $\widehat{\text{ARL}}_0 = 4$		
	0.25	—	0.8	0.5	3	0.6
	0.5	—	0.2	0.1	3	0.2
0.5	0.75	—	0.05	0.07	3	0.1
	1	—	0.01	0.05	3	0.05
	1.5	—	< 0.01	0.02	3	0.01
	2	—	< 0.01	0.02	3	< 0.01
	4	—	< 0.01	< 0.01	3	< 0.01
		se of $\widehat{\text{ARL}}_0 = 3$	se of $\widehat{\text{ARL}}_0 = 3$	se of $\widehat{\text{ARL}}_0 = 4$		
	0.25	—	1	0.7	3	0.9
	0.5	—	0.3	0.2	3	0.3
0.7	0.75	—	0.1	0.1	3	0.2
	1	—	0.04	0.08	3	0.1
	1.5	—	< 0.01	0.04	3	0.03
	2	—	< 0.01	0.03	3	0.01
	4	—	< 0.01	< 0.01	3	< 0.01
		se of $\widehat{\text{ARL}}_0 = 3$	se of $\widehat{\text{ARL}}_0 = 3$	se of $\widehat{\text{ARL}}_0 = 4$		
	0.25	2	2	2	3	2
	0.5	1	0.8	0.7	3	0.9
0.9	0.75	0.6	0.4	0.4	3	0.5
	1	0.3	0.2	0.2	3	0.3
	1.5	0.1	0.03	0.1	3	0.2
	2	0.04	< 0.01	0.07	3	0.08
	4	< 0.01	< 0.01	0.03	3	< 0.01

Table S4: Comparisons of four charts: R&W, tuned R&W, DFTC, and modified-Method-2 charts for an ARTA(1) process with exponential marginal distribution, $\rho_1 = 0$, 0.25, 0.5, 0.7, 0.9, and $L = 1000$ (The lowest $\widehat{\text{ARL}}_\delta$ is boxed.)

					Modified Method 2		
ρ_1	δ	R&W	Tuned R&W	DFTC	m	$\widehat{\text{ARL}}_0$	$\widehat{\text{ARL}}_\delta$
		$\widehat{\text{ARL}}_0 = 72.5$ (m=1)	$\widehat{\text{ARL}}_0 = 1006$ (m=100)	$\widehat{\text{ARL}}_0 = 999$ (m=1)			
	0.25	—	125	102	66	1013	117
	0.5	—	100	43	30	1011	43
0	0.75	—	100	28	30	1011	30
	1	—	100	20	30	1011	30
	1.5	—	100	13	30	1011	30
	2	—	100	10	30	1011	30
	4	—	100	5	30	1011	30
		$\widehat{\text{ARL}}_0 = 277$ (m=4)	$\widehat{\text{ARL}}_0 = 1006$ (m=110)	$\widehat{\text{ARL}}_0 = 1105$ (m=1)			
	0.25	—	169	149	89	1021	168
	0.5	—	110	65	37	1014	64
0.25	0.75	—	110	41	30	1019	35
	1	—	110	30	30	1019	30
	1.5	—	110	20	30	1019	30
	2	—	110	15	30	1019	30
	4	—	110	7	30	1019	30
		$\widehat{\text{ARL}}_0 = 468$ (m=9)	$\widehat{\text{ARL}}_0 = 1006$ (m=200)	$\widehat{\text{ARL}}_0 = 1220$ (m=1)			
	0.25	—	263	227	122	1021	250
	0.5	—	200	101	55	1025	100
0.5	0.75	—	200	64	31	1018	55
	1	—	200	47	30	1008	35
	1.5	—	200	31	30	1008	30
	2	—	200	23	30	1008	30
	4	—	200	11	30	1008	30
		$\widehat{\text{ARL}}_0 = 704$ (m=19)	$\widehat{\text{ARL}}_0 = 1006$ (m=250)	$\widehat{\text{ARL}}_0 = 1223$ (m=3)			
	0.25	—	376	334	163	1025	369
	0.5	—	252	159	79	1034	158
0.7	0.75	—	250	103	48	1045	90
	1	—	250	76	31	959	58
	1.5	—	250	50	30	944	32
	2	—	250	37	30	944	30
	4	—	250	19	30	944	30
		$\widehat{\text{ARL}}_0 = 1144$ (m=64)	$\widehat{\text{ARL}}_0 = 1006$ (m=350)	$\widehat{\text{ARL}}_0 = 1371$ (m=7)			
	0.25	687	671	613	236	1045	686
	0.5	391	394	329	146	1079	348
0.9	0.75	231	351	219	94	1092	212
	1	144	350	163	64	1147	144
	1.5	78	350	108	31	892	76
	2	64	350	80	30	876	39
	4	64	350	41	30	876	30

Table S5: Standard errors (se's) of $\widehat{\text{ARL}}_0$ and $\widehat{\text{ARL}}_\delta$ in Table S4 for ARTA(1) processes with exponential marginal distribution, $\rho_1 = 0$, 0.25, 0.5, 0.7, 0.9, and $L = 1000$

					Modified Method 2	
ρ_1	δ	R&W	Tuned R&W	DFTC	se of $\widehat{\text{ARL}}_0$	se of $\widehat{\text{ARL}}_\delta$
		se of $\widehat{\text{ARL}}_0 = 0.4$	se of $\widehat{\text{ARL}}_0 = 3$	se of $\widehat{\text{ARL}}_0 = 3$		
	0.25	—	0.2	0.2	4	0.3
	0.5	—	< 0.01	0.05	4	0.08
0	0.75	—	< 0.01	0.03	4	0.01
	1	—	< 0.01	0.02	4	< 0.01
	1.5	—	< 0.01	< 0.01	4	< 0.01
	2	—	< 0.01	< 0.01	4	< 0.01
	4	—	< 0.01	< 0.01	4	< 0.01
		se of $\widehat{\text{ARL}}_0 = 1$	se of $\widehat{\text{ARL}}_0 = 3$	se of $\widehat{\text{ARL}}_0 = 4$		
	0.25	—	0.4	0.3	4	0.4
	0.5	—	0.02	0.08	4	0.1
0.25	0.75	—	< 0.01	0.04	4	0.05
	1	—	< 0.01	0.03	4	0.01
	1.5	—	< 0.01	0.01	4	< 0.01
	2	—	< 0.01	< 0.01	4	< 0.01
	4	—	< 0.01	< 0.01	4	< 0.01
		se of $\widehat{\text{ARL}}_0 = 2$	se of $\widehat{\text{ARL}}_0 = 3$	se of $\widehat{\text{ARL}}_0 = 4$		
	0.25	—	0.5	0.5	4	0.6
	0.5	—	0.02	0.1	4	0.2
0.5	0.75	—	< 0.01	0.07	4	0.1
	1	—	< 0.01	0.05	4	0.05
	1.5	—	< 0.01	0.02	4	< 0.01
	2	—	< 0.01	0.02	4	< 0.01
	4	—	< 0.01	< 0.01	4	< 0.01
		se of $\widehat{\text{ARL}}_0 = 2$	se of $\widehat{\text{ARL}}_0 = 3$	se of $\widehat{\text{ARL}}_0 = 4$		
	0.25	—	0.8	0.7	4	1
	0.5	—	0.09	0.2	4	0.4
0.7	0.75	—	< 0.01	0.1	4	0.2
	1	—	< 0.01	0.08	3	0.1
	1.5	—	< 0.01	0.04	3	0.03
	2	—	< 0.01	0.03	3	< 0.01
	4	—	< 0.01	< 0.01	3	< 0.01
		se of $\widehat{\text{ARL}}_0 = 4$	se of $\widehat{\text{ARL}}_0 = 3$	se of $\widehat{\text{ARL}}_0 = 4$		
	0.25	2	2	1	4	2
	0.5	1	0.5	0.5	4	1
0.9	0.75	0.7	0.07	0.3	4	0.6
	1	0.4	< 0.01	0.2	4	0.4
	1.5	0.09	< 0.01	0.1	3	0.2
	2	< 0.01	< 0.01	0.07	3	0.07
	4	< 0.01	< 0.01	0.03	3	< 0.01